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ECONOMIC-MATHEMATICAL MODELING OF A MULTI-CRITERIA OPTIMIZATION MANAGEMENT PROBLEM OF A RETAIL UNIT OF A COMMERCIAL BANK

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The task to develop and use economic-mathematical models that increase the speed and quality of decision-making is becoming acute in terms of increasing competition at the banking service market, and in the consequences of the need for permanent increase of economic efficiency of banking institutions. Currently most of the decisions are made by banks in an expert way or on the basis of single calculations of economic efficiency of individual projects taking into account the available statistical data on the dynamics of macroeconomic processes. Problems of this kind are usually implemented on the basis of stochastic economic-mathematical models that are based on the knowledge about the probabilistic characteristics of the simulated parameters. For their construction it is necessary to observe rather harsh conditions on implementation of the above considered processes that it is difficult to realize in practice. Hereby, to solve a multi-criteria optimization management problem of a retail unit of a commercial bank we suggest using dynamic economic-mathematical models that include control actions and a vector criterion of quality. The tools under consideration focus on the increase of efficiency of decision-making in the field of the control of the staff amount, as well as in the sales system of a retail unit of a commercial bank. The stages of the creation of dynamic models and an algorithm for the solution of the vector optimization problem have been worked out. For the solution of the optimization problem of the management of a retail unit of a commercial bank we have made calculations for the set of practical tasks in the software environment Delphi 7. And the results obtained during the calculations have been illustrated. The analysis of the results has been made and the selection of an optimal solution has been explained. The suggested method for the vector optimization is characterized with significant practical application as it can be used as the basis for the development of systems to support management decision-making in various areas of the banking sector. Further works in the field will be devoted to the complication of the suggested tools by expanding the phase vector of the management system of a retail unit of a commercial bank by including additional quality criteria for the implementation of the management process under consideration, as well as through the solution of the problem of dynamic economic-mathematical modeling of the process of optimization of adaptive management of the similar process.

Keywords: dynamic economic-mathematical modeling, vector optimization problem, computer modeling, efficiency of a management system, control of the number of employees, system of retail sales, a commercial bank, a retail unit.

ЭКОНОМИКО-МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ МНОГОКРИТЕРИАЛЬНОЙ ОПТИМИЗАЦИОННОЙ ЗАДАЧИ УПРАВЛЕНИЯ РОЗНИЧНЫМ ПОДРАЗДЕЛЕНИЕМ КОММЕРЧЕСКОГО БАНКА

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В условиях усиления конкуренции на рынке банковских услуг и, как следствие, необходимости перманентного повышения экономической эффективности деятельности банковских организаций актуализируется задача формирования и использования экономико-математических моделей, позволяющих в значительной степени увеличить скорость и качество принимаемых управленческих решений. В настоящее время большинство таких решений банки принимают экспертным путем либо на основании разовых расчетов экономической эффективности отдельных проектов с учетом имеющихся статистических данных о динамике макроэкономических процессов. Задачи такого рода, как правило, реализуются на базе стохастических экономико-математических моделей, основанных на знании вероятностных характеристик моделируемых параметров. Для их построения необходимо соблюдать достаточно жесткие условия на реализацию рассматриваемых процессов, что трудно выполнить на практике. В связи с этим для решения многокритериальной оптимизационной задачи управления розничным подразделением коммерческого банка предлагается использовать динамические экономико-математические модели, в которые включены управляющие воздействия и векторный критерий качества. Предложенный инструментарий направлен на повышение эффективности принятия решений в области управления численностью персонала, а также системой продаж розничного подразделения коммерческого банка. Разработаны этапы создания динамической модели и алгоритм решения задачи векторной оптимизации. Для решения рассматриваемой оптимизационной задачи управления розничным подразделением коммерческого банка в программной среде *Delphi* 7 произведены расчеты для класса практических задач с иллюстрацией полученных результатов, проведен их анализ и выбрано оптимальное решение. Предложенный метод решения задачи векторной оптимизации имеет высокий потенциал практического применения в качестве основы разработки систем для поддержки принятия управленческих решений в различных сферах банковской деятельности. Перспективы исследования включают усложнение предложенного инструментария путем расширения фазового вектора системы управления розничным подразделением коммерческого банка посредством включения в нее дополнительных критериев качества реализации рассматриваемого процесса управления, в том числе за счет решения задачи динамического экономико-математического моделирования процесса оптимизации адаптивного управления аналогичным процессом.

Ключевые слова: динамическое экономико-математическое моделирование, задача векторной оптимизации, компьютерное моделирование, эффективность системы управления, управление численностью персонала, система розничных продаж, коммерческий банк, розничное подразделение.

Introduction

The issues related to the efficiency of bank management are significantly important. In terms of unstable financial behaviour of an external environment the managers of any bank considering the constantly changing "regulators" demands strive for risk minimisation and net operation profit growth of the business. At the same time the field is rapidly developing, modern technologies and e-channels of selling are being implemented. Competition in the field is still significant despite the participant amount decrease. In the response to constantly increasing demands of customers both in service rate and service quality representatives of the banking service market have to improve the activity rapidly in order to be constant operators at the banking service market. Micro-lending organizations and other companies providing financial service but that are not considered to be banks also compete with the latter. Constant strategy specification and tactical approaches to management in lending agencies are also considered to be competition tools. Marketing approaches for customers' attraction including advertisement also have a significant impact.

Management of the resources including personnel is an important trend to achieve significant efficiency of bank management. Considering rapid development of remote channels of service the necessity to maintain a chain of additional offices network regardless of whether they are leased or owned is still acute. In addition to the network maintenance the issues related to the upkeep of personnel working with costumers and their amount decrease are quite urgent [1–3]. The existing staff should work as efficient as possible, besides standards of sales should be involved, and deviation from the planned indicators should be controlled. As a rule decisions that determine the banking business development are made on the basis of expert opinion of the members of the board of the bank and other persons whom decision-making depends on. Decisions based on particular financial calculations of the activities are made rather seldom [4]. And the cost of the error in a poor decision is often very high, as it can lead to significant financial damage. In addition to possible losses the efficiency of the staff resources that were attracted from the market on the decision-making basis is rather discussible. Employees are not paid bonuses that they could have been paid when the planned indicators had been received in the event of erroneous decision to hire them as a result of the absence of the expected economic effect of their work. Quite often the staff has to perform functions unusual for their duties, look for opportunities for retraining, or change jobs. In case a decision on the massive reduction of the staff is made employees are dismissed, and are forced to register in the employment centres or move to less paid positions.

An information and analytical decision support system, which provides the access to quality data for their operational analysis significantly increases the efficiency of the banking process management [5–7]. The system quickly and efficiently assesses the consequences that are expected in case particular management or financial decisions are made and it helps to select the most efficient one [8; 9].

A management or financial decision support system includes a set of interrelated methods, software and hardware that automatically implement a set of data of various reports containing the necessary information for decision-making [10]. The most modern systems can offer various options for such solutions [11–13].

The application of economic-mathematical models and methods including machine learning technologies, neural networks for Big Data analysis and other tools makes the management of particular objects or banking activity process more efficient [14–17]. The use of the latest approaches allows us to obtain results to find solutions to the problems that were not taken into account in the original algorithms, as the system itself is trained over time on the results of banking activities. It should be noted that the global transition to the use of digital technologies is currently at an early stage of the development and concerns mainly only a part of banking processes (mass sales campaigns, identification of customer needs, etc.). At the same time, with regard to management decision-making related to strategic or short-term planning, these processes are in their infancy.

Thus, the issue related to the application of an economic-mathematical modeling to improve the speed and quality of management decisions in banking is significantly important.

The creation of an economic-mathematical model for the management of a retail unit of a commercial bank

The procedure for the dynamic economic-mathematical model creation in terms of the investigation of the optimization of the retail business management of a bank is described here. Similar mathematical models for economic systems are presented, for example, in the works [18–20]

We shall consider input data and peculiarities of management decision-making for a retail unit of a bank operation in case the number of employees and sales standards for their various roles change. Similar models for banking activities, only in a more simplified form, were considered in the works [21; 22].

The following symbols should be introduced to construct an economic-mathematical model for the decision-making:

n is a number of main "portfolio" products for individuals (e.g. mortgage, car loans, deposits for individuals, debit cards, etc.; $n \in \mathbf{N}$; where $\mathbf{N}$ is the set of all natural numbers here and further in the article);

m is a total number of employees' roles who dispose products for individuals (some roles have sales responsibilities for several types of products, others – only for one product; $m \in \mathbf{N}$);

$x(t) = \left(x_1(t), x_2(t), \dots, x_n(t)\right)' \in \mathbf{R}^n$ is a vector describing a portfolio volume for each bank product during the time interval t in thousands of roubles ($t \in \overline{0, T-1} = \{0,1,2, \dots T-1\}; T \in \mathbf{N}$), at which each the i^{th} coordinate $x_i(t)$ corresponds to the portfolio volume value of the i^{th} bank products type ($i \in \overline{1,n}$); here and further in the text for $k \in \mathbf{N}$, $\mathbf{R}^k$ is k – dimensional Euclidean vector space of column vectors;

$y(t) = \left(y_1(t), y_2(t), \dots, y_m(t)\right)' \in \mathbf{R}^m$ is a vector describing the quantity of the bank personnel roles during the t ($t \in \overline{0, T-1}$) time period, where each j^{th} coordinate $y_j(t)$ is the value of the employee headcount of the j^{th} position type ($j \in \overline{1,m}$) according to the staff schedule;

$A(t) = \left\| a_{ij}(t) \right\|, i \in \overline{1,n}, j \in \overline{1,m}$ is a matrix of monthly sales standards at the time interval t ($t \in \overline{0, T-1}$), $a_{ij}(t)$ is a normative amount of the soled products of the i^{th} type by the employee of the j^{th} role, in items ($i \in \overline{1,n}, j \in \overline{1,m}$);

$z = (z_1, z_2, \dots, z_n)' \in \mathbf{R}^n$ is a vector of labour costs per each product type sale considering the "sales funnel" (i.e. time spent on consulting all customers regardless of their decision to purchase the product), hr.;

$H = (h_1, h_2, \dots, h_n)' \in \mathbf{R}^n$ is a vector of portfolio amortization coefficients of each group (type) of the product per month (portfolio depreciation), %;

$S = (s_1, s_2, \dots, s_n)' \in \mathbf{R}^n$ is a vector of mean receipts of each product sold, RUB, thous.;

$u(t) = \left(u_1(t), u_2(t), \dots, u_m(t)\right)' \in \mathbf{R}^m$ is a vector entering a quantity of each staff role (amount of people) for t ($t \in \overline{0, T-1}$), where each j^{th} coordinate $u_j(t)$ is the value of the number of added staff units of employees of the j^{th} type of the role ($j \in \overline{1,m}$).

The dynamics of bank product portfolios for individuals will be described by a system of quasilinear recurrent equations below:

$$\begin{cases} y_j(t+1) = y_j(\tau_t) + u_j(\tau_t),\ u_j(0) = 0, \\ \tau_t = \tau \cdot E(\frac{t}{\tau}), \\ x_i(t+1) = x_i(t) - h_i \frac{x_i(t)}{100} + \\ + s_i \sum_{j=1}^{m} a_{ij}(t) \left[y_j(t) + u_j(t)\right], \\ t \in \overline{0, T-1}, i \in \overline{1,m}, j \in \overline{1,n}, \end{cases} \quad (1)$$

where the following symbols are used:

$x(t+1) = (x_1(t+1), x_2(t+1), \dots, x_n(t+1))' \in \mathbf{R}^n$ is a vector of bank product portfolio volumes during $(t+1)$ time period;

$y(t+1) = (y_1(t+1), y_2(t+1), \dots, y_m(t+1))' \in \mathbf{R}^m$ is a vector of quantity of the different roles of employees (number of people) at the time interval $(t+1)$;

$\tau \in \boldsymbol{N}: \tau \le T$;

$E: \mathbf{R}^1 \to \mathbf{Z}$ is a function of a number aliquot.

The system (1) under consideration allows us to simulate the dynamics of a multistep process of managing (control) the structure of the portfolio of banking products for individuals depending on the given initial conditions and on the choice of specific management decisions: changes in the number of employees and the establishment of sales standards for them.

A vector entering the number of employees at a bank retail unit $u(t) = \left(u_1(t), u_2(t), \dots, u_m(t)\right)' \in \mathbf{R}^m$ and the sales standard matrix $A(t) =$ $i \in \overline{1,n}$, $j \in \overline{1,m}$ at the time interval t ($t \in \overline{0, T-1}$) are the control actions in the system, that needs the following constraints to be introduced:

$$u(t) \in U_1\left(t, y(t-1), y(t)\right) \subset \mathbf{R}^m,$$

$$U_1(t) = \left\{u(t): u(t)\left\{u^{(1)}(t), u^{(2)}(t), \dots, u^{(N_t)}(t)\right\}\right\} \subset \mathbf{R}^m, N_t \in \mathbf{N},$$

$$y(t) = (y_1(t), y_2(t), \dots, y_m(t))' \in \mathbf{R}^m,$$

$(\forall j \in \overline{1,m}) \wedge (\forall k \in \overline{1,N_t})$: $|u_j^{(k)}(t)| \leq 0{,}1 \cdot y_j(t-1)$;
$\sum_{j=1}^{m} y_j(t) \leq 10000$; (2)
$A(t) \in A_1(t) \subset \boldsymbol{R}^{n\times m}$, $A_1(t) = \{A(t): A(t) \in \in \{A^{(1)}(t), A^{(2)}(t), \dots, A^{(M_t)}(t)\} \subset \mathbf{R}^{n\times m}, M_t \in \mathbf{N}$,
$\forall k \in \overline{1,M_t}$: $A^{(k)}(t) = \left\| a_{ij}^{(k)}(t) \right\|$,
$i \in \overline{1,n}, j \in \overline{1,m}$;
$\Big((\forall i \in \overline{1,n}) \wedge (\forall j \in \overline{1,m})$: $a_{ij}^{(k)}(t) \geq \geq 0 \Big) \wedge \left(T_{min} \leq \sum_{i=1}^{n} \left[a_{ij}^{(k)}(t) z_i \right] \leq T_{max} \right)$;
$\Big((\forall i \in \overline{1,n}) \wedge (\forall j \in \overline{1,m})$: $a_{ij}^{(k)}(t) \geq 0 \Big) \wedge (\forall a_{ij}^{(k)}(t) > 0)$:
$a_{ij}^{(k)}(t) z_i \geq 0{,}1 \cdot \sum_{i=1}^{n} \left[a_{ij}^{(k)}(t) z_i \right], z_i \in \mathbf{R}^1 \}$, (3)

where T_{min} is a minimum value of time to sell banking products per month, min., T_{max} is a standard value of working hours per month, min.; N_1 is an amount of possible control actions $u\,(t)$ at the time interval t ($N_1 \in \mathbf{N}$); M_1 is an amount of possible matrixes $A(t)$ of sales standards at the time interval t ($M_1 \in \mathbf{N}$).

At the same time the following phase restrictions must be introduced for all $t \in \overline{0, T-1}$:

$$x(t) \in X_1(t), y(t) \in Y_1(t),$$
$$X_1(t) = \left\{ x(t): x(t) = (x_1(t), x_2(t), \dots, \ x_n(t))' \in \in \mathbf{R}^n, \forall i \in \overline{1,n}: \ x_i(t) \geq 0 \right\},$$
$$Y_1(t) = \left\{ y(t): y(t) = (y_1(t), y_2(t), \dots, y_m(t))' \in \in \mathbf{R}^m, \forall i \in \overline{1,m}: \ y_i(t) \geq 0 \right\}. \quad (4)$$

Within the task under consideration we establish a restriction that control actions can be applied no more than once every 6 months.

We shall describe a set of quality criteria that can be used for the assessment of the results of possible implementations of the process that is described by the system (1) – (4).

We shall catalogue a pair $\bar{u}(\cdot) = = \{u(\cdot), A(\cdot)\} \in \overline{\boldsymbol{U}}(\cdot)$, that makes a valid program control at the time interval $\overline{0,T}$, where $u(\cdot) = \{u(t)\}_{t\epsilon\overline{0,T-1}}$; $A(\cdot) = = \{A(t)\}_{t\epsilon\overline{0,T-1}}$; $\forall t \in \overline{0,T-1}$: $\overline{\boldsymbol{U}}(t) = = U_1(t, y(t-1), y(t)) \times A_1(t)$,
$\overline{\boldsymbol{U}}(0) = U_1(0) \times A_1(0)$; $\overline{\boldsymbol{U}}(\cdot) = \left\{ \overline{\boldsymbol{U}}(t) \right\}_{t\epsilon\overline{0,T-1}}$
are multitudes of all valid program control actions at the time interval $\overline{0,T}$ in the dynamic system (1)–(4) under consideration.

We shall designate $\bar{x}(T) = = \varphi_{\overline{0,T}}(T;\ \bar{x}(0), \bar{u}(\cdot))$ a final state (at the time interval *T*) of the trajectory $x(\cdot) = \{x(t)\}_{t\epsilon\overline{0,T}}$ of the phase vector $\bar{x}(t) = \{x(t),\ y(t)\}' \in \in \mathbf{R}^{m+n}$ $(t \in \overline{0,T})$ of the system (1) – (4), that describes the dynamics of the optimization process under consideration at a time interval $\overline{0,T}$, that corresponds to the set $\{\bar{x}(0), \bar{u}(\cdot)\} \in \overline{X}(0) \times \overline{\boldsymbol{U}}(\cdot)$, where $\overline{X}(0) = = \{x(0)\} \times \{y(0)\}$.

Then for all possible variants of the sets $\{\bar{x}(0), \bar{u}(\cdot)\} \in \overline{X}(0) \times \overline{\boldsymbol{U}}(\cdot)$ the quality criterion of the control process in the system (1) – (4), that describes the dynamics of the process of optimization of complex program control of the number of employees in a bank retail unit and the system of sales during the time interval $\overline{0,T}$ can be estimated by the following terminal functions (the process quality indicators) [23; 24].

A functional of a bank retail unit profits

We shall add additional parameters to the model under consideration:

$v = (v_1, v_2, \dots, v_m)' \in \mathbf{R}^m$ is a vector of the salary size of the employees of each roal, ₽, thous.;

$r = (r_1, r_2, \dots, r_n)' \in \mathbf{R}^n$ is a vector of interest and transfer income rate for each portfolio type, % per annum;

$c = (c_1, c_2, \dots, c_n)' \in \mathbf{R}^n$ is a vector of interest and transfer income rate of expences for each portfolio type, % per annum;

q is a value of administrative and economic costs per month, excluding salary costs, ₽, thous.

Then the target function $\mathbf{\Phi}_{0,T}^{(1)}(\ \bar{x}(0), \bar{u}(\cdot))$ for the optimization of the complex program control of the number of employees at a bank retail unit and the system of sales at the time interval $\overline{0,T}$ is calculated:

$$\mathbf{\Phi}_{0,T}^{(1)}(\ \bar{x}(0), \bar{u}(\cdot)) = p(T) = = \sum_{t=1}^{T} \Big(\frac{1}{12*100} \cdot \sum_{i=1}^{n} (x_i(t) + x_i(t-1)) \cdot \frac{1}{2} \cdot (r_i - c_i) - \sum_{j=1}^{m} v_j\, y_j(t) - q \Big) = = F_{0,T}^{(1)}\left(\varphi_{\overline{0,T}}(T;\ \bar{x}(0), \bar{u}(\cdot)) \right) = F_{0,T}^{(1)}(\ \bar{x}(T)), \quad (5)$$

where $p(t)$ is a bank profit value for all types of products individuals cumulative, ₽, thous., during the time interval t ($t \in \overline{1,T}$); $p(0) = 0$.

The maximum value is an optimal value for the functional.

Cost Income Ratio* (*CIR*) *functional – the ratio of the transaction costs to the transaction income of a bank retail unit

The target function $\mathbf{\Phi}^{(2)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$ for the optimization of a complex program control of the number of employees at a bank retail unit and of a sales system that demonstrates the Cost Income Ratio (CIR) value for the time interval $\overline{0,T}$, is calculated according to the equation:

$$\mathbf{\Phi}^{(2)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot)) = CIR(T) =$$
$$= \frac{\sum_{t=1}^{T}(\sum_{j=1}^{m} v_j\, y_j(t) + q)}{\sum_{t=1}^{T}(\frac{1}{12*100}\cdot\sum_{i=1}^{n}(x_i(t) + x_i(t-1))\cdot\frac{1}{2}\cdot(r_i - c_i))} =$$
$$= F^{(2)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}(T;\,\overline{x}(0),\overline{u}(\cdot))\right) = F^{(2)}_{\overline{0,T}}(\overline{x}(T)), \qquad (6)$$

where $CIR(t)$ is a ration of transaction costs to transaction income of a bank retail unit in cumulative, %, for the time interval $t\,(t \in \overline{1,T})$; $CIR(0) = 0$.

The minimum value is optimal for the criterion.

Functional of the share increase at the individuals' outside funds market

Let $m(t)$ to be the individuals' outside funds volume for the time interval t, ₽, thous.. The market volume forecast for future is made by a bank macroeconomic research centre on the basis of official forecasts of macroeconomic indicators.

The bank share growth at the individuals' outside fund market is calculated according to the formula:

$$\mathbf{\Phi}^{(3)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot)) = b(T) = \frac{L(T)}{m(T)} - \frac{L(0)}{m(0)} =$$
$$= \frac{\sum_{i\in B} x_i(T)}{m(T)} - \frac{\sum_{i\in B} x_i(0)}{m(0)} =$$
$$= F^{(3)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}(T;\,\overline{x}(0),\overline{u}(\cdot))\right) = F^{(3)}_{\overline{0,T}}(\overline{x}(T)),$$
$$B = \left\{b : b = (b_1, b_2, \dots, b_c)' \in \mathbf{R}^c, c \le\right.$$
$$\left.\le n, \forall i \in \overline{1,c} :\ 0 < b_i \le n, b_i \in \mathbf{N}\right\}, \qquad (7)$$

where $L(T)$ is a volume of bank retail liabilities, ₽, thous., for the time interval $\overline{0,T}$; $b(T)$ is the bank share growth at the individuals' outside fund market, %, for the time interval $\overline{0,T}$; B is a set of natural numbers, that the indices of the $x(t)$ vector coordinates correspond to. The coordinates determines the parameters of the bank liabilities portfolio ($B \subset \mathbf{N}$).

An optimal value for the functional is a maximum value.

The feature list (functionality) that characterises the quality of the process under consideration may vary according to the strategic objectives of the financial institution. Strategic goals are determined by a bank management and depend on the external economic situation, on the behaviour of competitors, and on internal achievements or problem areas of the company. A list of three target functions that includes the income of a bank retail unit, *CIR* and a share growth at the individuals' outside funds market is considered as a part of the task solution.

On the basis of the target functionals $\mathbf{\Phi}^{(1)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$, $\mathbf{\Phi}^{(2)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$ and $\mathbf{\Phi}^{(3)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$ described by the ratios (5) – (7) to determine the quality of optimization process of the complex program control of the number of employees at a bank retail unit and the sales system we introduce a generalized target function $\mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$. The values of the latter for all possible valid implementations of the sets $\{\overline{x}(0),\overline{u}(\cdot)\} \in \in \overline{X}(0) \times \overline{\boldsymbol{U}}(\cdot)$ at the time interval $\overline{0,T}$ are calculated in accordance to the correlation:

$$\mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot)) = \lambda_1\cdot\mathbf{\Phi}^{(1)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot)) -$$
$$-\lambda_2\cdot\mathbf{\Phi}^{(2)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot)) +$$
$$+\lambda_3\cdot\mathbf{\Phi}^{(3)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot)) =$$
$$= \lambda_1\cdot F^{(1)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}(T;\,\overline{x}(0),\overline{u}(\cdot))\right) -$$
$$-\lambda_2\cdot F^{(2)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}(T;\,\overline{x}(0),\overline{u}(\cdot))\right) +$$
$$+\lambda_3\cdot F^{(3)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}(T;\overline{x}(0),\overline{u}(\cdot))\right) =$$
$$= \lambda_1\cdot F^{(1)}_{\overline{0,T}}(\overline{x}(T)) - \lambda_2\cdot F^{(2)}_{\overline{0,T}}(\overline{x}(T)) +$$
$$+\lambda_3\cdot F^{(3)}_{\overline{0,T}}(\overline{x}(T)) =$$
$$= \lambda_1\cdot F^{(1)}_{\overline{0,T}}(x(T), y(T)) -$$
$$-\lambda_2\cdot F^{(2)}_{\overline{0,T}}(x(T), y(T)) +$$
$$+\lambda_3\cdot F^{(3)}_{\overline{0,T}}(x(T), y(T)) =$$
$$= F(\overline{x}(T)) = F(x(T), y(T)),$$
$$\forall i \in \overline{1,3}: \lambda_i \ge 0;\ \sum_{i=1}^{3}\lambda_i = 1. \qquad (8)$$

The generalized target function $\mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$ is a scalar convolution of vector quality criteria $\mathbf{\Phi}^{(1)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$, $\mathbf{\Phi}^{(2)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$ and $\mathbf{\Phi}^{(3)}_{\overline{0,T}}(\overline{x}(0),\overline{u}(\cdot))$. Scalarization of vector target functions is described in [25] in details. Moreover, weighting coefficients $\lambda_i, i \in \overline{1,3}$ demonstrate the importance of each quality criteria in terms of the impact on the selection of the retail management strategy of a bank. The coefficients may be varied and simulation results may be analyzed. Usually their values are determined on the basis of the opinions of a person or a group of persons making management decision. In addition, different methods of expert estimations are used for processing and generalization of opinions. They are the method of comparative importance of criteria, brainstorming, Delphi method, method of scenarios, etc.

Thus, the optimization of the complex program control of the number of employees at a bank retail unit and of the sales system can be expressed in the following way.

For the optimisation process of the complex program control of the number of employees at a bank retail unit and of the sales system that is considered at a particular time interval $\overline{0,T}$ and described by the discrete dynamic economic-mathematical model (1) – (8) and for the particular initial phase vector $\overline{x}(0) = \{x(0), y(0)\}' \in \mathbf{R}^{m+n}$ these two valid software controls $\overline{u}^{(e)}(\cdot) = \{u^{(e)}(\cdot), A^{(e)}(\cdot)\} \in \overline{\boldsymbol{U}}(\cdot)$ should be distinguished at the particular time period in order the linear terminal target function value $\mathbf{\Phi}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot))$ determined by the correlation (8) was the maximum in comparison with all other values valid for this function corresponding to other pairs of valid software controls, i.e. the optimality condition described below was executed:

$$
\begin{aligned}
&\boldsymbol{\Phi}^{(e)}(T) = \boldsymbol{\Phi}_{\overline{0,T}}\left(\overline{x}(0), \overline{u}^{(e)}(\cdot)\right) = \\
&= \max_{\overline{u}(\cdot)=\{u(\cdot),A(\cdot)\}\in\overline{\boldsymbol{U}}(\cdot)} \boldsymbol{\Phi}_{\overline{0,T}}\left(\overline{x}(0), \overline{u}(\cdot)\right) = \\
&= \max_{\overline{u}(\cdot)=\{u(\cdot),A(\cdot)\}\in\overline{\boldsymbol{U}}(\cdot)} \{\lambda_1 \cdot \mathbf{\Phi}^{(1)}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot)) - \\
&-\lambda_2 \cdot \mathbf{\Phi}^{(2)}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot)) + \\
&+\lambda_3 \cdot \mathbf{\Phi}^{(3)}_{\overline{0,T}}(\overline{x}(0), \overline{u}(\cdot))\} = \\
&= \max_{\overline{u}(\cdot)=(u(\cdot),A(\cdot))\in\overline{\boldsymbol{U}}(\cdot)} \{\lambda_1 \cdot \\
&\cdot F^{(1)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}(T;\ \overline{x}(0), \overline{u}(\cdot))\right) - \\
&-\lambda_2 \cdot F^{(2)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}(T; \overline{x}(0), \overline{u}(\cdot))\right) + \\
&+\lambda_3 \cdot F^{(3)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}(T; \overline{x}(0), \overline{u}(\cdot))\right)\} = \\
&= \lambda_1 \cdot F^{(1)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}\left(T; \overline{x}(0), \overline{u}^{(e)}(\cdot)\right)\right) - \\
&-\lambda_2 \cdot F^{(2)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}\left(T; \overline{x}(0), \overline{u}^{(e)}(\cdot)\right)\right) + \\
&+\lambda_3 \cdot F^{(3)}_{\overline{0,T}}\left(\varphi_{\overline{0,T}}\left(T; \overline{x}(0), \overline{u}^{(e)}(\cdot)\right)\right) = \\
&= \lambda_1 \cdot F^{(1)}_{\overline{0,T}}\left(\bar{x}^{(e)}(T)\right) - \\
&-\lambda_2 \cdot F^{(2)}_{\overline{0,T}}\left(\bar{x}^{(e)}(T)\right) + \lambda_3 \cdot F^{(3)}_{\overline{0,T}}\left(\bar{x}^{(e)}(T)\right) = \\
&= \lambda_1 \cdot F^{(1)}_{\overline{0,T}}\left(x^{(e)}(T), y^{(e)}(T)\right) - \\
&-\lambda_2 \cdot F^{(2)}_{\overline{0,T}}\left(x^{(e)}(T), y^{(e)}(T)\right) + \\
&+\lambda_3 \cdot F^{(3)}_{\overline{0,T}}\left(x^{(e)}(T), y^{(e)}(T)\right) = \\
&= F\left(\bar{x}^{(e)}(T)\right) = F\left(x^{(e)}(T), y^{(e)}(T)\right) = \\
&= F\left(\bar{x}^{(e)}(T)\right) = F\left(x^{(e)}(T), y^{(e)}(T)\right) = \boldsymbol{F}^{(e)}(T); \\
&\forall i \in \overline{1,3}: \lambda_i \geq 0;\ \textstyle\sum_{i=1}^{3} \lambda_i = 1, \qquad (9)
\end{aligned}
$$

where $\bar{x}^{(e)}(T) = \varphi_{\overline{0,T}}\left(T; \overline{x}(0), \overline{u}^{(e)}(\cdot)\right) = \{x^{(e)}(T), y^{(e)}(T)\}$.

The output results of the optimisation process of the complex program control of the number of employees at a bank retail unit are a set of data $\{u^{(e)}(\cdot), A^{(e)}(\cdot), \boldsymbol{F}^{(e)}(T)\}$, where $u^{(e)}(\cdot) = \{u^{(e)}(t)\}_{t\epsilon\overline{0,T-1}}$ is an array of optimal values of the vector of entering of the number of employees at the bank retail unit, $A^{(e)}(\cdot) = \{A^{(e)}(t)\}_{t\epsilon\overline{0,T-1}}$ is an array of optimal values of sales standards matrices of banking products by employees at a bank retail unit, $\boldsymbol{F}^{(e)}(T)$ is an optimal value of the quality functional at the final time period T.

Thus, the procedure for the optimal solution of the task under consideration can be used on the basis of the obtained formalized description of the economic-mathematical model of the bank retail unit management.

The algorithm for the optimal software management of the bank retail unit

We shall consider an algorithm for the optimal program control of the number of employees at a bank retail unit and of the system of sales they make in a particular practical case study that is considered to be a particular case of the general economic-mathematical model (1) – (9).

Given $n = 8$, $m = 7$ and the parameters of the system (1) – (9) are described at the time interval $\overline{0,T}$ during the time period $t(t \in \overline{0,T})$ with the data:

$\tau = 6$, i.e. the number of employees at a bank retail unit may change at 6 months intervals – not often than ones every six months;

$x(t) = \left(x_1(t), x_2(t), \dots, x_8(t)\right)' \in \mathbf{R}^8$;

$x_1(t)$ is a consumer loan balance (CL);

$x_2(t)$ is a mortgage loan balance (ML);

$x_3(t)$ is a car loan balance (CarL);

$x_4(t)$ is a credit card balance (CC);

$x_5(t)$ is a balance at individuals' time deposits (D);

$x_6(t)$ is a balance at personal debit bank card accounts (DC);

$x_7(t)$ is a balance at individuals' securities accounts (S);

$x_8(t)$ is a balance at salary card accounts (SC).

An array of the values of the coordinates of the vector $x(t)$ that characterise the portfolio of bank liabilities includes the following values: $B = \{5, 6, 7, 8\}$.

The vector of the number of employees:

$y(t) = \left(y_1(t), y_2(t), \dots, y_7(t)\right)' \in \mathbf{R}^7$;

$y_1(t)$ is the number of sales managers;

$y_2(t)$ is a mortgage lending manager headcount;

$y_3(t)$ is a direct sales specialist headcount;

$y_4(t)$ is a headcount of senior specialists in direct sales;

$y_5(t)$ is a headcount of salary project managers;

$y_6(t)$ is a headcount of managers working with partners;

$y_7(t)$ is a number of specialists working with private individuals.

The initial state of the phase vector $\overline{x}(0) = \{x(0),\ y(0)\}' \in \mathbf{R}^{15}$ at the time moment $t = 0$ is known.

Constraints (2) on the discrete control value $u(t) = \left(u_1(t), u_2(t), \dots, u_7(t)\right)' \in \mathbf{R}^7$ for each $t \in \overline{0, T-1}$ is suggested to be:

$u(t) \in U_1(t, y(t-1), y(t)) = \{u(t): u(t) \in$
$\in \left\{u^{(1)}(t), u^{(2)}(t), u^{(3)}(t)\right\} \subset \mathbf{R}^7, \forall\, k \in \overline{1,3}$,
$u^{(k)}(t) = (u_1^{(k)}(t), u_2^{(k)}(t), \dots, u_7^{(k)}(t)) \in \mathbf{R}^7$,
$y(t) = (y_1(t), y_2(t), \dots, y_7(t))' \in \mathbf{R}^7$,
$y(t-1) = (y_1(t-1), y_2(t-1), \dots,$
$y_7(t-1))' \in \mathbf{R}^7$,
$\forall j \in \overline{1,7}, u_j^{(k)}(t) \in \{-u_j^{*(k)}(t); 0; u_j^{*(k)}(t)\}$,
$u_j^{*(k)}(t) = 0{,}1 \cdot y_j(t-1)$,

$$\sum_{j=1}^{7} y_j(t) \le 10000,$$

$y_j(t) = y_j(t-1) + u_j(t-1)$,
$t \ge 2: y_j(t-1) = y_j(t-2) + u_j(t-2)\}$,

i.e. $N_t = 3$, for all $t \in \overline{0, T-1}$. Herewith the control $u(t)$ determines the first multiplier of the generalized control action $\overline{u}(t) =$ $= \{u(t), A(t)\} \in \overline{\boldsymbol{U}}(t)$, where $u(t) = u^{(k)}(t) \in$ $\in U_1(t)$ for all $t \in \overline{0, T-1}$ and $k \in \overline{1,3}$, and a single initial control value $u(0) = u^{(k)}(0) =$ $= \mathbf{0}_7 \in U_1(0)$, т.е. $U_1(0) = \{u(0)\}$ – a singleton array is suggested to be specified for each $k \in \overline{1,3}$, where $\mathbf{0}_7 = (0,0, \dots, 0)'$ is a zero vector of the $\mathbf{R}^7$ space.

In accordance to constraint (3) a sequence of matrices $A_1(t) = \{A(t): A(t) \in$ $\in \{A^{(1)}(t), A^{(2)}(t), A^{(3)}(t)\}$ is specified. The latter includes three matrices, i.e. given $M_t = 3$, for all $t \in \overline{0, T-1}$. There each corresponding matrix $A^{(k)}(t) \in A_1(t)$ $(k \in \overline{1,3})$ at each time period t $(t \in \overline{0, T-1})$ determines the second multiplier of the generalized control action $\overline{u}(t) = \{u(t), A(t)\}$ $\in \overline{\boldsymbol{U}}(t)$, where $A(t) = A^{(k)}(t)$. It means that the program control can be simulated under different matrices. At the same time it assumes that a single initial value of the sales standard matrix $A(0) = A^{(k)}(0) \in A_1(0)$ is set for each $k \in \overline{1,3}$.

Then the modeling algorithm for the solution of the optimal program control of the number of employees at a bank retail unit in the presence of vector target function of the type (8) may be presented as the implementation of the following sequence of actions.

Step 0. Initial data generation.

0.1. The natural number $T \in \mathbf{N}$ is introduced that determines an optimisation period of the process under consideration control.

0.2. The initial value of the phase vector $\bar{x}(0) = \{x(0), y(0)\}' \in \mathbf{R}^{15}$ is formed for $t = 0$.

0.3. The vector $u(0) = (u_1(0), u_2(0), \dots, u_7(0))' \in \mathbf{R}^7$, that determines the singleton $U_1(0) = \{u(0)\}$ in accordance to constraint (2) is formed for $t = 0$.

0.4. The matrix $A(0)$ that determines the singleton $A_1(0) = \{A(0)\}$ in accordance to the constraint (3) is formed for $t = 0$.

0.5. A set of real numbers $(\lambda_1, \lambda_2, \lambda_3)$, $\forall i \in \overline{1,3}: \lambda_i \geq 0;\ \sum_{i=1}^{3} \lambda_i = 1$, that determines the coefficients of the vector functional $\mathbf{\Phi}_{\overline{0,T}}$ is introduced. The values of the latter are formed by the ratio (8).

0.6. The actual numeric value $\boldsymbol{F}^{(e)}(T) = -10^{10}$ is formed.

Step 1. Generation of a set of possible control actions.

A finite set of all possible program controls at the time interval $\overline{0,T}$ is formed:

$\overline{\boldsymbol{U}}(\cdot) = \{\bar{u}(\cdot): \bar{u}(\cdot) = \{u(\cdot), A(\cdot)\}$,
$u(\cdot) = \{u(t)\}_{t \in \overline{0,T-1}}, A(\cdot) = \{A(t)\}_{t \in \overline{0,T-1}}$,
$\forall t \in \overline{0, T-1}$,
$u(t) = (u_1(t), u_2(t), \dots, u_7(t))' \in \mathbf{R}^7$,
$A(t) = \|a_{ij}(t)\|_{\substack{i \in \overline{1,7} \\ j \in \overline{1,8}}} \in \mathbf{R}^{7\times 8}$,
$y(t) = \left(y_1(t), y_2(t), \dots, y_7(t)\right)' \in \mathbf{R}^7$,
$y(t-1) = (y_1(t-1), y_2(t-1), \dots, y_7(t-1))' \in \mathbf{R}^7$,
$\forall j \in \overline{1,7}, u_j(t) \in \{-u_j^*(t); 0; u_j^*(t)\}$,
$u_j^*(t) = 0{,}1 \cdot y_j(t-1), \sum_{j=1}^{7} y_j(t) \leq 10000$,
$y_j(t) = y_j(t-1) + u_j(t-1)$,
$t \geq 2: y_j(t-1) = y_j(t-2) + u_j(t-2)$,
$A(t) \in A_1(t) = \{A^{(1)}(t), A^{(2)}(t), A^{(3)}(t)\}\}$,
$\overline{\boldsymbol{U}}(0) = U_1(0) \times A_1(0) = \{u(0), A(0)\}$,

that consists of $K_t = 9^{7\times(T-1)}$ elements of $\bar{u}^{(k)}(\cdot), k \in \overline{1, K_t}$.

Step 2. Generation of a final phase vector of the system.

2.1. Start of cycle 1 by the integer variable $k \in \overline{1, K_T}$.

2.2. The control action $\bar{u}^{(k)}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\} \in \overline{\boldsymbol{U}}(\cdot)$, where $\bar{u}^{(k)}(0) = \{u(0), A(0)\}$ is formed.

2.3. The value of the phase vector $\bar{x}^{(k)}(T) = \{x^{(k)}(T), y^{(k)}(T)\} = \varphi_{\overline{0,T}}(T; \bar{x}^{(k)}(0), \bar{u}^{(k)}(\cdot))$, where $\bar{x}^{(k)}(0) = \{x^{(k)}(0), y^{(k)}(0)\}' = \{x(0), y(0)\}' \in \mathbf{R}^{15}$ is calculated in accordance to the dynamics equations (1).

Step 3. Generation of the value of the vector functional.

3.1. For the phase vector $\bar{x}^{(k)}(T) = \{x^{(k)}(T), y^{(k)}(T)\} = \varphi_{\overline{0,T}}(T; \bar{x}^{(k)}(0), \bar{u}^{(k)}(\cdot))$ that corresponds to the k^{th} program control $\bar{u}^{(k)}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\} \in \overline{\boldsymbol{U}}(\cdot)$ in accordance to the ratio (8) the value of the functionality $F\left(\bar{x}^{(k)}(T)\right)$ is calculated according to the formula:

$$F\left(\bar{x}^{(k)}(T)\right) = F\left(x^{(k)}(T), y^{(k)}(T)\right) = \lambda_1 \cdot F_{\overline{0,T}}^{(1)}\left(x^{(k)}(T), y^{(k)}(T)\right) - \lambda_2 \cdot F_{\overline{0,T}}^{(2)}\left(x^{(k)}(T), y^{(k)}(T)\right) + \lambda_3 \cdot F_{\overline{0,T}}^{(3)}\left(x^{(k)}(T), y^{(k)}(T)\right).$$

3.2. If $F\left(\bar{x}^{(k)}(T)\right) > \boldsymbol{F}^{(e)}(T)$, then proceed to section 3.3, if not, to section 3.5.

3.3. The values of the real variable are generated: $\boldsymbol{F}^{(e)}(T) = F\left(\bar{x}^{(k)}(T)\right)$.

3.4. The array of the control actions is generated: $\bar{u}^{(e)}(\cdot) = \bar{u}^{(k)}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\} \in \overline{\boldsymbol{U}}(\cdot)$.

3.5. The end of cycle 1 on the integer variable $k \in \overline{1, K_T}$.

Step 4. The representation of the modeling results.

The following parameters are calculated as a result of the algorithm step implementation:

4.1. The actual number $\boldsymbol{F}^{(e)}(T)$ is an optimal value of the vector quality criterion for the process under consideration.

4.2. The actual array $\bar{u}^{(e)}(\cdot) = \{u^{(e)}(\cdot),\ A^{(e)}(\cdot)\} \in \bar{\boldsymbol{U}}(\cdot)$ is an optimal program control for the process under consideration, where $u^{(e)}(\cdot) = \{u^{(e)}(t)\}_{t\in\overline{0,T-1}}$; $A^{(e)}(\cdot) = \{A^{(e)}(t)\}_{t\in\overline{0,T-1}}$; $\bar{\boldsymbol{U}}(\cdot) = \{\bar{\boldsymbol{U}}(t)\}_{t\in\overline{0,T-1}}$.

4.3. The actual array $\bar{x}^{(e)}(\cdot) = \{x^{(e)}(\cdot), y^{(e)}(\cdot)\} = \{x^{(e)}(t)\}_{t\epsilon\overline{0,T-1}}$, $\{y^{(e)}(t)\}_{t\epsilon\overline{0,T-1}}\}$ is an optimal trajectory for the process under consideration, i.e. $\bar{x}^{(e)}(\cdot) = \varphi_{\overline{0,T}}\left(\cdot; \bar{x}^{(e)}(0), \bar{u}^{(e)}(\cdot)\right), \bar{x}^{(e)}(0) = \{x^{(e)}(0), y^{(e)}(0)\}' \in \mathbf{R}^{15}$.

4.4. The results are presented in a user-friendly form, such as graphs or tables.

The end of the algorithm.

It is significant that the elements $\bar{u}^{(e)}(\cdot) = \{u^{(e)}(\cdot), A^{(e)}(\cdot)\} \in \bar{\boldsymbol{U}}(\cdot)$ and $\boldsymbol{F}^{(e)}(T)$ that were obtained with the algorithms under consideration meet the optimality condition (9), i.e. they are the solution for the problem of optimization of the complex program control of the number of employees at a bank retail unit and the sales system if a vector target function of the form (8) is presented.

The algorithm we have suggested to solve the task of the optimal program control of a retail unit of a commercial bank in the presence of a vector target function can be implemented in the form of a computer modeling system. We shall consider the suggested implementation of the algorithm in a particular case study.

Generation of the initial data of computer modelling of an optimal program control of a retail unit of a commercial bank and the results of its implementation

We shall consider the task of optimal software program control of the number of employees and sales of a bank retail unit, described by the relations (1) – (9) with the following initial conditions:

The initial condition (if *t*=0) of the phase vector $\bar{x}(0) = \{x(0), y(0)\}' \in \mathbf{R}^{15}$ and of the sales standards matrix $A(0)$ is known. The sequence of matrices $\{A(t)\}_{t\epsilon\overline{0,T-1}}$ is suggested to be set for the control process under consideration. The matrix $A(t)$ corresponding to the time period t ($t \in \overline{0, T-1}$) in the sequence of matrices determines the second multiplier of the control action. It means that the process of program control under different sets of such matrix sequences can be simulated.

At the time interval *t*=0 the sales standard matrix $A(0) = A^{(k)}(0) \in A_1(t),\ k \in \overline{1,3}$ has the following specific form:

Matrix $A(0)$

$$\begin{pmatrix} 6 & 8 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 19 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

The initial values of the following parameters are set:

$$z = (200,\ 500,\ 200,\ 50,\ 50,\ 280,\ 40,\ 35);$$
$$H = (10.75,\ 5.0,\ 13.0,\ 6.2,\ 5.0,\ 7.0,\ 8.2,\ 3.0);$$
$$S = (150,\ 1827,\ 720,\ 38,\ 280, 68,\ 340,\ 50);$$
$$u\,(0) = (0, 0, 0, 0, 0, 0, 0);$$
$$x\,(0) = (140\,000\,000,\ 160\,000\,000,\ 35\,000\,000,\ 42\,000\,000,\ 300\,000\,000,\ 70\,000\,000,\ 140\,000\,000,\ 80\,000\,000);$$
$$y\,(0) = (1636,\ 215,\ 250,\ 155,\ 42,\ 35,\ 1830);$$
$$v = (60, 70, 40, 56, 52, 58, 40);$$
$$r = (13.0,\ 10.4,\ 12.0,\ 15.2,\ 10.0,\ 6.0,\ 9.2,\ 9.6);$$
$$c = (10.0,\ 10.0,\ 10.0,\ 10.0,\ 8.0,\ 0.1,\ 9.0,\ 0.1),$$
$$q = 900\,000;\ m(0) = 1\,100\,000\,000,$$
$$m(18) = 1\,400\,000\,000.$$

And the following constraints:

$$T_{min} = 9600;\ T_{max} = 11040;$$
$$(i = 2) \wedge (\forall i \in \overline{5,7}): a_{i1} = 0;$$
$$(\,i = 5)\ \wedge (i = 7): a_{i2} = 0;$$
$$(\,\forall i \in \overline{2,5}) \wedge (i = 7): a_{i3} = 0;$$
$$(i = 2) \wedge (\forall i \in \overline{5,6}):\ a_{i4} = 0,$$
$$a_{i5} = 0, a_{i6} = 0;\ \forall i \in \overline{2,6}:\ a_{i7} = 0;$$
$$(\,\forall i \in \overline{1,4}) \wedge (\forall i \in \overline{6,7}): a_{i8} = 0.$$

Let $\lambda = \{0.2, 0.2, 0.6\}$.

The algorithm under consideration to solve the task of optimal program terminal control is implemented for the class of practical tasks in a form of a computer simulation system in the software environment Delphi 7.

Then we describe the results of computer simulation.

The sales standards matrix $A(t), t \in \overline{0, T-1}$ is a control action in the dynamic system (1) – (9) under consideration and its possible values were generated in accordance to the existing constraints.

Taking into account the existing restrictions on the sales of particular banking products by different categories of employees (in accordance with the specialization of employees), as well as the data on the profitability of the respective portfolios and labour costs for the sale of the products, we consider 3 options for the values of sales standards for each product per month presented in the corresponding matrices $A^{(k)}(t)$ $k \in \overline{1,3}$ with the maximum values of sales standards for each product (column). The forms of each of these matrices are presented below.

Matrix $A^{(1)}(t)$ (CL)

$$\begin{pmatrix} 19 & 3 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 30 & 3 & 0 & 23 & 23 & 4 & 0 & 0 \\ 30 & 3 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 3 & 47 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

Matrix $A^{(2)}(t)$ (ML)

$$\begin{pmatrix} 6 & 8 & 6 & 23 & 23 & 4 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 6 & 12 & 0 & 23 & 23 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 19 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

Matrix $A^{(3)}(t)$ (DC)

$$\begin{pmatrix} 6 & 3 & 6 & 23 & 23 & 12 & 28 & 0 \\ 0 & 22 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 3 & 0 & 23 & 23 & 21 & 0 & 0 \\ 6 & 3 & 0 & 23 & 23 & 21 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 315 \\ 0 & 3 & 47 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23 & 23 & 27 & 28 & 0 \end{pmatrix}$$

The modelling process for the solution of the task of an optimal program control of the system (1) – (9) was considered at the time interval $\overline{0,T} = \overline{0,18}$ with an opportunity to change the possible control action $\bar{u}(t) = \{u(t), A(t)\}$ at the time intervals $t \in \{0,6,12\} \subset \overline{0,18}$, and the following conditions were suggested to be met:

$$\bar{u}(0) = \{u(0), A(0)\};\ \forall t \in \overline{0,5}: \bar{u}(t) = \bar{u}(0) = \{u(0), A(0)\};\ \forall t \in \overline{6,11}: \bar{u}(t) = \bar{u}(6) = \{u(6), A(6)\} \in \boldsymbol{\bar{U}}(6);\ \forall t \in \overline{12,17}: \bar{u}(t) = \bar{u}(12) = \{u(12), A(12)\} \in \boldsymbol{\bar{U}}(12).$$

Then under the specified initial data and restrictions for the control actions the array of all possible program controls is formed. The array is determined by the set $\boldsymbol{\bar{U}}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\}_{k\epsilon\overline{1,K_{18}}}$ that consists of $K_{18} = 9^{7\times 2} = 9^{14}$ elements $\bar{u}^{(k)}(\cdot) = \{u^{(k)}(\cdot), A^{(k)}(\cdot)\}, k\epsilon\overline{1,K_{18}}$, and the array of all possible phase trajectories $\bar{x}^{(k)}(\cdot) = \{x^{(k)}(\cdot), y^{(k)}(\cdot)\} = \varphi_{\overline{0.18}}(\cdot; \bar{x}^{(k)}(0), \bar{u}^{(k)}(\cdot))$, $k\epsilon\overline{1,K_{18}}$, of the dynamic system under consideration corresponds to it. The set is also consists of the $K_{18} = 9^{14}$ elements, where $\forall k\epsilon\overline{1,K_{18}}$: $\bar{u}^{(k)}(\cdot) = \{\bar{u}^{(k)}(t)\}_{t\epsilon\overline{0,18}}$, $u^{(k)}(\cdot) = \{u^{(k)}(t)\}_{t\epsilon\overline{0,18}}$, $A^{(k)}(\cdot) = \{A^{(k)}(t)\}_{t\epsilon\overline{0,18}}$; $\forall t\epsilon\overline{0,18}: \bar{u}^{(k)}(t) = \{u^{(k)}(t), A^{(k)}(t)\} \in \boldsymbol{\bar{U}}(t)$.

The array of possible final phase conditions of the system of the form: $\bar{x}^{(k)}(18) = \{x^{(k)}(18), y^{(k)}(18)\} = \varphi_{\overline{0.18}}\left(\cdot; \bar{x}^{(k)}(0), \bar{u}^{(k)}(\cdot)\right)$ corresponds to the set of all possible trajectories. At the same time single a priory given initial sales standards, i.e. $\forall k\epsilon\overline{1,3}: A^{(k)}(0) = A(0)$ are suggested to be for each trajectory at the control interval $\overline{0,18}$. The values of particular and generalized quality criteria of the process under consideration are calculated for each k^{th} final value of the phase vector $\bar{x}^{(k)}(18) = \{x^{(k)}(18), y^{(k)}(18)\} = \varphi_{\overline{0,18}}(18; \bar{x}^{(k)}(0), \bar{u}^{(k)}(\cdot))$, $k\epsilon\overline{1,K_{18}}$ in accordance to the formulae (5) – (8).

Then an optimal program control $u^{(e)}(\cdot) = \{u^{(e)}(\cdot), A^{(e)}(\cdot)\} \in \boldsymbol{\bar{U}}(\cdot) = \{u^{(e)}(\cdot), A^{(e)}(\cdot)\}_{t\epsilon\overline{0.18}} \in \boldsymbol{\bar{U}}(\cdot)$, the corresponding trajectory $\bar{x}^{(e)}(\cdot) = \{x^{(e)}(\cdot), y^{(e)}(\cdot)\} = \varphi_{\overline{0.18}}\left(18; \bar{x}^{(e)}(0), \bar{u}^{(e)}(\cdot)\right)$, the real number $\boldsymbol{F}^{(e)}(T)$ that is an optimal value of the vector quality criterion for the control process under consideration and an optimal value of the final phase vector $\bar{x}^{(e)}(18) = \{x^{(e)}(18), y^{(e)}(18)\} = \varphi_{\overline{0.18}}(18; \bar{x}^{(e)}(0), \bar{u}^{(e)}(\cdot))$, where $\bar{x}^{(e)}(0) = \bar{x}(0) = \{x(0), y(0)\}$ are calculated on the basis of the algorithm for the modelling of the optimal program control of the number of employees and sales at a bank retail unit in the presence of a vector target function of the form (8).

The computer modelling results are presented in Tables 1, 2, and in Fig. 1.

Table 1

Optimal implementation of the control of the number of employees

$u^{(e)}(t)$ \ t	**0**	**6**	**12**
$u_1{}^{(e)}(t)$	0	163	179
$u_2{}^{(e)}(t)$	0	-21	-19
$u_3{}^{(e)}(t)$	0	25	27
$u_4{}^{(e)}(t)$	0	15	17
$u_5{}^{(e)}(t)$	0	4	4
$u_6{}^{(e)}(t)$	0	3	3
$u_7{}^{(e)}(t)$	0	183	201

Table 2

Optimal implementation of sales standards control

t	**0**	**6**	**12**
$A^{(e)}(t)$	$A^2(ML)$	$A^1(CL)$	$A^3(DC)$

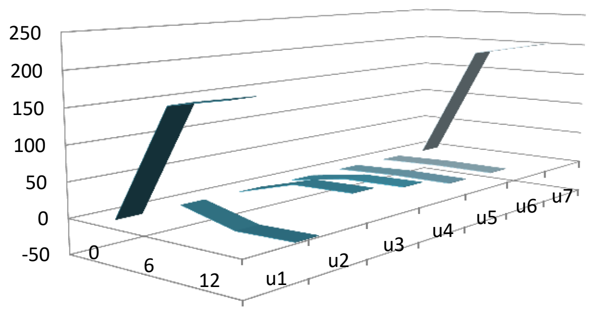

Fig. 1. Control action implementation of the number of employees $u^{(e)}(t)$

The generalized criterion value in the case is $\boldsymbol{\Phi}^{(e)}(18) = \boldsymbol{\Phi}_{\overline{0,18}}\left(\overline{x}(0), \overline{u}^{(e)}(\cdot)\right) = 0{,}7966$.

Fig. 2, 3 and tables 3, 4 present possible implementation of the control of the employees' number equal to $u^{(1)}(t)$, $u^{(2)}(t)$ when the optimal control implementation of the sales standards is $A^{(e)}(t)$.

Table 3

Control action implementation of the number of employees $u^{(1)}(t)$

$u^{(1)}(t)$ \ t	**0**	**6**	**12**
$u_1{}^{(1)}(t)$	0	-163	-147
$u_2{}^{(1)}(t)$	0	-21	-19
$u_3{}^{(1)}(t)$	0	-25	-22
$u_4{}^{(1)}(t)$	0	-15	-14
$u_5{}^{(1)}(t)$	0	-4	-3
$u_6{}^{(1)}(t)$	0	-3	-3
$u_7{}^{(1)}(t)$	0	-183	-164

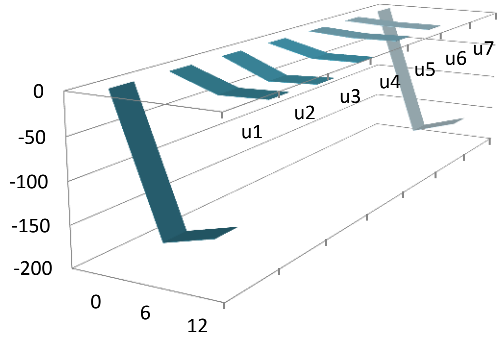

Fig. 2. Implementation of the control action of the number of employees $u^{(1)}(t)$

The value of the generalized criterion is $\boldsymbol{\Phi}^{(1)}(18) = \boldsymbol{\Phi}_{\overline{0,18}}\left(\overline{x}(0), \overline{u}^{(1)}(\cdot)\right) = 0{,}1504$.

Table 4

Implementation of the control action of the number of employees $u^{(2)}(t)$

$u^{(2)}(t)$ \ t	**0**	**6**	**12**
$u_1{}^{(2)}(t)$	0	0	163
$u_2{}^{(2)}(t)$	0	0	21
$u_3{}^{(2)}(t)$	0	0	25
$u_4{}^{(2)}(t)$	0	0	15
$u_5{}^{(2)}(t)$	0	0	4
$u_6{}^{(2)}(t)$	0	0	3
$u_7{}^{(2)}(t)$	0	0	183

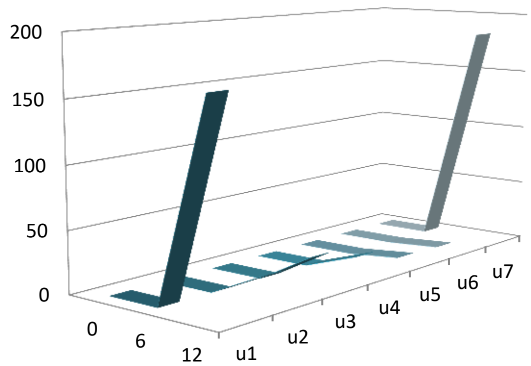

Fig. 3. Implementation of the control action of the number of employees $u^{(2)}(t)$

The value of the generalized criterion is $\boldsymbol{\Phi}^{(2)}(18) = \boldsymbol{\Phi}_{\overline{0,18}}\left(\overline{x}(0), \overline{u}^{(2)}(\cdot)\right) = 0{,}5643$.

Thus, as it has been mentioned before a trajectory corresponding to the optimal program control for which the value of the generalized criterion of the quality of the above considered process implementation at the final moment of time is the maximum in comparison with similar values for other trajectories corresponding to admissible program controls is considered to be the most efficient to form the dynamic system (1) – (9). Control when the number of mortgage lending managers is decreasing during the time periods $t = 6$ and $t = 12$, and the amount of other employees is increasing is the most efficient for the dynamic system with the parameters values specified before. When sales standards are established, maximum attention during $t = 6$ is paid to consumer credits and during $t = 12$ – to debit cards. In this case the value of the generalized criterion is 0.7966 for $t = 18$, and that is the absolute maximum.

The main results obtained during the implementation of the economic and mathematical modelling method under consideration that has been used to solve the problem of optimal program control of the number of employees and sales of a retail unit of a commercial bank are listed below:

– the existing methods used to increase the speed and quality of the implemented management decisions in banking sector indicating the importance of the issue under consideration and the demand for a dynamic economic and mathematical model for the research of the management of a retail unit of a commercial bank have been assessed;

– a list of steps to achieve the necessary result by developing a dynamic economic and mathematical model to simulate the process of managing the number of employees and sales of a retail unit of a commercial bank has been described;

– an algorithm for the task of dynamic economic-mathematical modeling to optimise the program control of employees amount and sales rate of a retail unit commercial bank has been suggested;

– the selection of the most efficient solution for the task under consideration has been based on the developed computer modeling system in the Delphi 7 and on the modeling results.

Conclusions

A new dynamic economic-mathematical model to improve the management of the personnel amount and the sales of a retail unit of a commercial bank has been described in the article. The application of the model allows us to solve one of the tasks of the process – the formation of an optimal amount of the employees, as well as the determination of the necessary sales standards to provide the best values of the quality criteria of the management process.

Complication of the suggested economic-mathematical model by the expanding the phase vector of the system and the inclusion of additional quality criteria for the implementation of the process under the investigation is a promising direction.

In addition the developed dynamic economic-mathematical model may become the basis for the creation, implementation and application of an integrated information and analytical system to support management decision-making in the banking sector. Taking into account the fact that the problem of vector optimization of the process of control of the number of employees and sales of the retail unit of a commercial bank that has been considered in the research is included in the scope of activities of both personnel services of a bank and financial departments, planning departments, as well as business departments, a wide practical application of the developed model in various activities of the banking organization is possible. At the same time, the use of dynamic economic and mathematical models significantly increases the efficiency of a commercial bank, accelerating and optimizing the management decision-making process. It makes a credit institution more competitive and flexible in relation to the changes in the external environment.

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